NYSED EMSC
New York State Education Department
Elementary, Middle, Secondary and Continuing Education

# High School Mathematics Course Descriptions

The three new high school mathematics courses (Integrated Algebra, Geometry, Algebra & Trigonometry) are built around five process strands: Problem Solving, Reasoning and Proof, Communication, Connections, and Representation as well as five content strands: Number Sense and Operations, Algebra, Geometry, Measurement, and Statistics and Probability. Within these courses, students will be expected to make connections between the verbal, numerical, algebraic, and geometric representations of problem situations. These courses will require students to apply and adapt a selection of strategies and algorithms to solve a variety of problems. It is expected that these strategies and algorithms will be implemented using both traditional and technological tools. A brief description of the three courses follows.

***Integrated Algebra*** is the first mathematics course in the high school. The integrated algebra course set forth here is not the algebra of 30 years ago. The focal point of this course is the algebra content strand. Algebra provides tools and ways of thinking that are necessary for solving problems in a wide variety of disciplines, such as science, business, social sciences, fine arts, and technology. This course will assist students in developing skills and processes to be applied using a variety of techniques to successfully solve problems in a variety of settings. Problem situations may result in all types of linear equations in one variable, quadratic functions with integral coefficients and roots as well as absolute value and exponential functions. Coordinate geometry will be integrated into the investigation of these functions allowing students to make connections between their analytical and geometrical representations. Problem situations resulting in systems of equations will also be presented. Alternative solution methods should be given equal value within the strategies used for problem solving. For example, a matrix solution to a system of equations is just as valid as a graphical solution or an algebraic algorithm such as elimination. Measurement within a problem-solving context will include calculating rates using appropriate units and converting within measurement systems. Data analysis including measures of central tendency and visual representations of data will be studied. An understanding of correlation and causation will be developed and reasonable lines of best fit will be used to make predictions. Students will solve problem situations requiring right triangle trigonometry. Elementary probability theory will be used to determine the probability of events including independent, dependent and mutually exclusive events.

***Geometry*** is intended to be the second course in mathematics for high school students. There is no other school mathematics course that offers students the opportunity to act as mathematicians. Within this course, students will have the opportunity to make conjectures about geometric situations and prove in a variety of ways, both formal and informal, that their conclusion follows logically from their hypothesis. This course is meant to employ an integrated approach to the study of geometric relationships. Integrating synthetic, transformational, and coordinate approaches to geometry, students will justify geometric relationships and properties of geometric figures. Congruence and similarity of triangles will be established using appropriate theorems. Transformations including rotations, reflections, translations, and glide reflections and coordinate geometry will be used to establish and verify geometric relationships. A major emphasis of this course is to allow students to investigate geometric situations. Properties of triangles, quadrilaterals, and circles should receive particular attention. It is intended that students will use the traditional tools of compass and straightedge as well as dynamic geometry software that models these tools more efficiently and accurately, to assist in these investigations. Geometry is meant to lead students to an understanding that reasoning and proof are fundamental aspects of mathematics and something that sets it apart from the other sciences.

***Algebra 2 and Trigonometry*** is the capstone course of the three units of credit required for a Regents diploma. This course is a continuation and extension of the two courses that preceded it. While developing the algebraic techniques that will be required of those students that continue their study of mathematics, this course is also intended to continue developing alternative solution strategies and algorithms. For example, technology can provide to many students the means to address a problem situation to which they might not otherwise have access. Within this course, the number system will be extended to include imaginary and complex numbers. The families of functions to be studied will include polynomial, absolute value, radical, trigonometric, exponential, and logarithmic functions. Problem situations involving direct and indirect variation will be solved. Problems resulting in systems of equations will be solved graphically and algebraically. Algebraic techniques will be developed to facilitate rewriting mathematical expressions into multiple equivalent forms. Data analysis will be extended to include measures of dispersion and the analysis of regression that model functions studied throughout this course. Associated correlation coefficients will be determined, using technology tools and interpreted as a measure of strength of the relationship. Arithmetic and geometric sequences will be expressed in multiple forms, and arithmetic and geometric series will be evaluated. Binomial experiments will provide the basis for the study of probability theory and the normal probability distribution will be analyzed and used as an approximation for these binomial experiments. Right triangle trigonometry will be expanded to include the investigation of circular functions. Problem situations requiring the use of trigonometric equations and identities will also be investigated.

## Introduction

In recent years, much study and research have been carried out to discover ways in which students learn. This has led to student-centered classrooms where students are actively engaged in the learning process. Therefore, students must be presented with a classroom where both their intellectual engagement and physical engagement are of a high level. Teacher/student discourse and student activity should require high level thinking involving not just simple knowledge and understanding, but the more complex levels of thinking when students are required to analyze, synthesize, and justify mathematical ideas and relationships. This is sometimes referred to as *rigor*. *Rigor* can be thought of as the way in which students acquire knowledge.

A second dimension that must be considered in the classroom is *relevance*. *Relevance* refers to the application of knowledge. A mathematics classroom must include *pure mathematics* teaching and learning, but students should also have the opportunity to make application of the mathematics they learn. These applications may be in the discipline of mathematics itself, in other disciplines that the students study, and in real-life situations. However, real-life situations should be natural, not contrived. A school's curriculum, a teacher's instruction, and student assessment should all reflect high levels of both rigor and relevance.

## HIGH SCHOOL CROSSWALK

### Structural Organization

| 1996 Mathematics Standard | 1999 Core Curriculum | 2005 Mathematics Standard and Core Curriculum |
|---|---|---|
| Performance Indicators for:<br>• Commencement level<br>• Four-year Sequence | Performance Indicators for:<br>Two Regents Exams<br>• Math A<br>• Math B | Performance Indicators for:<br>Three Courses with a Regents Exam for Each Course<br>• Integrated Algebra<br>• Geometry<br>• Algebra 2 and Trigonometry |

### Comparison of 1999 Seven Key Ideas and 2005 Process and Content Strands

| 1999 Key Ideas | 2005 Process and Content Strands |
|---|---|
| • Broad in scope and transcend the various branches of mathematics (arithmetic, number theory, algebra, geometry, etc.).<br>• A column listing each performance indicator.<br>• A separate listing of Assessment Examples and Classroom Ideas for each Performance Indicator in Math A and Classroom Ideas for each Performance Indicator in Math B.<br>• Processes of mathematics (problem solving, communication, etc.) are, for the most part, included in the narrative of the document rather than having performance indicators to describe them. | • Process and Content Strands are aligned to the National Council of Teachers of Mathematics Standards.<br>• The processes of mathematics as well as the content of mathematics have performance indicators.<br>• Performance indicators are clearly delineated and more specific than in the 1996 learning standard and 1999 Core Curriculum. |

## Performance Indicator Organization

<table>
<tr><th>1996 Mathematics Standard and<br>1999 Core Curriculum</th><th>2005 Mathematics Standard and<br>2005 Core Curriculum</th></tr>
<tr><td>1996 Mathematics Standard<br>Seven Key Ideas<br>Mathematical Reasoning<br>Number and Numeration<br>Operations<br>Modeling and Multiple Representation<br>Measurement<br>Uncertainty<br>Patterns and Functions<br><br>Performance indicators are organized under the seven key ideas. A set of sample tasks are indicated for each standard to provide specificity to the performance indicators.</td><td rowspan="2">2005 Mathematics Standard<br><br>Five Process Strands<br>Problem Solving<br>Reasoning and Proof<br>Communication<br>Connections<br>Representation<br><br>Five Content Strands<br>Number Sense and Operations<br>lgebra<br>Geometry<br>Measurement<br>Statistics and Probability<br><br>Performance indicators are organized under major understandings within content and process strands. Content performance indicators are separated into bands within each of the content strands.</td></tr>
<tr><td>1999 Mathematics Core Curriculum<br>Seven Key Ideas<br>Mathematics Reasoning<br>Number and Numeration<br>Operations<br>Modeling and Multiple Representation<br>Measurement<br>Uncertainty<br>Patterns and Functions<br><br>Performance indicators are organized under the seven key ideas and contain an “includes” column to add specificity to the performance indicators.</td></tr>
</table>

## Performance Indicator Alignment

Although the 2005 mathematics standard is different in structure from the previous standard documents, there are many similarities as indicated in the degree of alignment shown in the crosswalk. By listing performance indicators for both process and content strands, the 2005 standard places as much emphasis on instruction as on curriculum. The performance indicators for the process strands stress instruction – how students learn and how teachers teach mathematics - and what mathematics is as a discipline. On the other hand, the performance indicators for the content strands stress curriculum – what students need to know and be able to do. The crosswalk included in this document primarily compares the content performance indicators of the 2005 Content Strands with those of the 1999 Core Curriculum. This in no way should be interpreted to mean that the process strands are of less importance than the content strands. They are both equally important. The performance indicators for the process strands were not included in the crosswalk because they basically did not have a counterpart in Standard 3 of the 1999 Core Curriculum. They are one of the major additions to the 2005 mathematics standard.

The following performance indicators from Math A and Math B have no counterpart in the content strands of the 2005 Core Curriculum at the high school level because they are included in the PreK-8 portion of the 2005 mathematics standard. This does not mean that they are not to be addressed at the high school level.

**Math A**

***Number and Numeration***

2B Recognize the order of real numbers

***Operations***

3C Recognize and identify symmetry and transformations on figures

***Modeling/Multiple Representation***

4C Use transformations in the coordinate plane

**Math B**

***Number and Numeration***

2B Recognize the order of real numbers
2C Apply the properties of the real numbers to various subsets of numbers

The following performance indicators from Math B have no direct counterpart in the content strands of the 2005 Core Curriculum at the high school level but are stressed within the five process strands.

***Problem Solving***

4A Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs

***Representation***

4C Choose appropriate representations to facilitate the solving of a problem

***Connections***

7G Model real-world situations with the appropriate function

The following performance indicators from Math A and Math B do not have any direct counterpart in the 2005 Standard. This does not mean, however, that they are not addressed in the 2005 Standard. There is a comment for each performance indicator with an explanation as to how it is addressed in the 2005 Standard.

**Math A**

***Measurement***

5F Apply proportions to scale drawings and direct variation

Although not specifically stated, students would be expected to make these applications of proportions during their study of proportions in Math 7, Math 8, Algebra, and Geometry.

5I Use geometric relationships in relevant measurement problems involving geometric concepts

Although not specifically stated, students should continually use geometric relationships as they are involved in the measurement and geometry strands of the standard.

***Patterns/Functions***

7D Model real-world situations with the appropriate function

Students should continually model real-world situations with the appropriate function while making connections in the *Connections* strand.

**Math B**

***Number and Numeration***

2D Recognize the hierarchy of the complex number system

Teachers should continually have students look at the development of our number system and see that each set of numbers is a subset of other sets; i.e., our number system develops in the following way: counting numbers, whole numbers, integers, rational numbers, irrational numbers, complex numbers and that each time a new type of number is introduced it has all the previous numbers as subsets. This is implied in the 2005 standard, but not specifically stated as a performance indicator.

***Modeling/Multiple Representation***

4D Develop meaning for basic conic sections

This is alluded to in G.G.1-G.G.7 (equations of conic sections) with basic meanings implied. Student activities should engage them in the basic meanings.

4G Represent graphically the sum and difference of two complex numbers

A2.N.9 requires operations with complex numbers, but the graphic sum and difference of two complex numbers is not specifically indicated. Although not specifically stated, it is expected that teachers would have students model the arithmetic operations graphically.

4H Model quadratic inequalities both algebraically and graphically

This performance indicator is a pre-requisite for the performance indicators in the new standard that deal with the solution of inequalities, but is not directly addressed.

4I Model the composition of transformations

Could be covered in the representation strand, but it is not directly stated there.

4K Use polynomial, trigonometric, and exponential functions to model real-world relationships

Students are expected to use polynomial, trigonometric, and exponential functions, but the standard does not mention application in real-world settings. Applications in real-world settings should be a part of the *Connections* strand, but applications should be natural, not contrived.

4N Use graphing utilities to create and explore geometric and algebraic models

Technology tools are implied to be fully integrated into the mathematics curriculum as described by the new standard. Hence, the direct statement of this performance indicator is unnecessary.

***Measurement***

5B Understand error in measurement and its consequence on subsequent calculations

Addressed somewhat in performance indicator A.M..3. Relative error in measuring square and cubic units when error occurs in linear measure. No mention is made of consequence on subsequent calculations. Teachers would be expected to discuss this in the application of the performance indicator.

***Patterns/Functions***

7C Translate among the verbal descriptions, tables, equations, and graphic forms of functions

Implied in several performance indicators, but never directly addressed. Teachers should continually show multiple representations throughout the entire curriculum.

7O Apply the ideas of symmetry in sketching and analyzing graphs of functions

Implied, but not specifically stated. Teachers would have students do this as they are engaged in activities involving the sketching and analyzing of graphs of functions.

## Content Performance Indicators Chart

The chart below depicts the number of performance indicators in the content strands for each of the three high school courses.

| **Number of Performance Indicators for Each Course** | | | | |
|---|---|---|---|---|
| **Content Strand** | **Algebra** | **Geometry** | **Algebra 2 and Trigonometry** | **Total** |
| **Number Sense and Operations** | 8 | 0 | 10 | **18** |
| **Algebra** | 45 | 0 | 77 | **122** |
| **Geometry** | 10 | 74 | 0 | **84** |
| **Measurement** | 3 | 0 | 2 | **5** |
| **Statistics and Probability** | 23 | 0 | 16 | **39** |
| **TOTAL** | **89** | **74** | **105** | **268** |